Mathematical Intimacy within Blended and Face-to-face Learning Environments

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Introduction

Oftentimes, learning mathematics involves using technology as a conscious attempt to understand the material in a new and accessible mode, to increase performance, and to expand one's knowledge regardless of the level of training. While comparative assessments of students' performances in blended and face-to-face environments are essential, the authors of this paper analyze students' mathematical intimacy and flow experiences, as well as their confidence and perseverance while learning mathematics in two different settings. Are students more engaged in problem solving, more inclined to experience joy, excitement and affection, more confident, and more persistent while doing mathematics in blended or in traditional learning environments? The authors of this paper aim to answer the question from the perspective of using MyMathLab, a Pearson based online course, within a blended teaching and learning environment for Algebra and Trigonometry, a large first year university mathematics course. This paper aims to analyze and interpret students' mathematical intimacy, confidence and perseverance in these two different learning environments.

MyMathLab is an online course designed by Pearson Education Canada as an accompaniment to its Algebra and Trigonometry textbook. MyMathLab is built on the MathXL platform, Pearson's online homework and assessment system and is accessed via CourseCompass, the Pearson online learning environment. University professors can choose MyMathLab for use throughout the whole course, or just some topics within the course. MyMathLab offers instructors and students a remarkable selection of course materials that range from a large database of exercises to multimedia resources, such as video lectures, animations, and an electronic version of the textbook. Instructors are not constrained to draw on the existing database; new items can be added. Practice exercises regenerate automatically for an indefinite number of times, thus offering students the opportunity to rehearse each math problem. To aid comprehension of mathematics concepts students can use the interactive solution guide and worked examples accompanying each exercise in the database. Students receive instant feedback upon solving each exercise. Moreover, MyMathLab offers an online grade book, which automatically registers students' homework results and gives instructors control over computing final marks.

Implementing new technologies is especially significant at the undergraduate level where students encounter a wide range of definitions, theorems and proofs that lay the foundation for more sophisticated mathematical thinking. Students' abilities to interpret, analyze, retrieve and use different mathematics concepts become crucial for future work in science. To improve students' performance, reduce high failure rates, and to create long-term sustainable teaching and learning strategies for large mathematics classes, MyMathLab was used for teaching the Algebra and Trigonometry course at one comprehensive Canadian University. MyMathLab was used during two classes in consecutive semesters: first with a class of 26 and then with a class of 127 students. In addition to the face-to-face teaching format, written assignments, midterm and final examinations, students were required to complete 10 MyMathLab quizzes each semester. A variety of factors contribute to students' achievements, thus making comparisons difficult, but based on the assessment scores taken prior to enrolment in the course and on the average final grade scores it is believed that the group using MyMathLab made greater progress than students who did not (Kondratieva & Radu, 2008). A snapshot of the research study testing the effectiveness of this environment shows that the percentage of students who received As increased from 12.9 percent at the time when the course was taught without MyMathLab to 15.4 percent at the time when MyMathLab was incorporated in the course. As well, the percentage of students receiving Bs increased from 19.3 percent to 26.9 percent (Kondratieva & Radu, 2008). This snapshot was taken during the first round of implementation of MyMathLab. During the subsequent semester, the picture was even clearer, because not only the percentage of students who received As increased from 12.6 percent to 18.8 percent, the course average of those receiving As increased from 85 percent to 88 percent. Furthermore, this report underlined that structural support coordinating MyMathLab exercises, in-class instruction and assessments practices could help students in achieving better math results. It predicated future usage of tighter support for every topic in the course, and a higher correlation between MyMathLab and in-class instruction.

Subsequently, MyMathLab was used within a blended learning environment, where the instructor combined traditional teaching methods and computer-mediated instruction strategies as part of classroom teaching. The instructor had the opportunity to monitor, control, adjust and match the blended teaching process with the weekly online homework offered through the online laboratory created to assist students in their mathematics e-journey. Again, students were required to complete 10 MyMathLab quizzes. While it is apparent that students' performances improve (Kondratieva & Radu, 2008), it makes sense to analyze students' emotional structures in blended and face-to-face environments, because how much success they eventually have in mathematics is intimately related to both the cognitive and the affective processes that characterize their thinking and problem solving in the subject (McLeod & Adams, 1989; Goldin, 2008). If the emotional tone of mathematical learning is integrally related to how mathematical information is perceived, processed, stored or retrieved, the potential value of studying the impacts of this learning could be essential. And this leads to one significant research question: Do students' mathematical intimacy and its positive by-products, namely confidence and persistence vary within blended and face-to-face environments?

In recent years, mathematics education researchers have started to pay attention to the role affective elements play in doing mathematics. Largely portrayed as encompassing emotions, beliefs, attitudes and values/morals/ethics, the affective domain is of primary concern for mathematicians and mathematics educators since it plays a fundamental role in the development and long-term appreciation of mathematics knowledge. Mathematics education researchers concerned with the learning of mathematics highlighted the importance of emotions in learning and problem solving performance (DeBellis, 1999; DeBellis & Goldin, 1997; McLeod & Adams, 1989). The emotive aspects of knowing could influence one's acquisition of mathematics knowledge. Affect, viewed as a representational system interacts with the cognitive representation systems, such as verbal, imagistic, formal notational and executive control (Goldin, 1987; 1988). As a representational system affect has a huge ability to encode as well as trade the affective information while interacting with other representational systems. Such exchange system is essential to "mathematical understanding and problem-solving performance" (DeBellis & Goldin, 2006, p. 133). Building on analogies with cognitive structures, affect as a representational system includes affective structures such as values, beliefs, attitudes and pathways of emotional feeling (DeBellis & Goldin, 2006). As a representational system, affect includes changing states of emotional feelings during mathematics problem solving, also known as local affect, as well as more permanent and stable constructs, known as global affect (DeBellis & Goldin, 2006). Situated within the context of local affect, mathematical intimacy is an affective structure that carries emotional meaning and weight for students.

Based on psychological research, mathematical intimacy is defined as a form of intimacy that consists of two components: intimate interactions and intimate relationships (DeBellis, 1998; Prager, 1995). A series of intimate mathematical interactions build up intimate relationships. Thus, the core of this affective structure lies with the intimate interactions, which are characterized by intimate mathematical behaviours and intimate mathematical experiences. According to DeBellis (1998), examples of intimate mathematical behaviours include "the distance a problem solver places between himself and his work, cradling his work, temporary loss of hearing external noises because he is so focused and consumed by the interaction, and hesitation in sharing mathematical solutions" (p. 437). Intimate mathematical experiences incorporate "positive feelings and perceptions of understanding which a problem solver incurs while solving a problem or thinking about a mathematical concept" (p. 437). Examples include warmth, passion, time suspension, vulnerability, loyalty, and positive emotions such as joy, excitement, affection, elation, or amusement. But, beyond this organized structure, experiencing mathematical intimacy is equivalent to being highly engaged in problem solving, having a warm-hearted dialogue with various math concepts, analyzing and comprehending its most inner structures, or creating a close bond with mathematics. Nevertheless, mathematical intimacy does not represent a guarantee to a "positive long-term relationship with mathematics" (DeBellis & Goldin, 2006, p. 138), as mathematics problem solvers could feel betrayed in intimacy. This occurs when they experience unpleasant emotions, "unexpected mathematical outcomes, failures, negative reactions from loved ones, rebuke from a trusted teacher, or scorn from peers" (DeBellis & Goldin, 1999, p. 252). The possibility of experiencing betrayal is explained through the vulnerability aspect of intimacy, and could be encountered by students and professors alike. The above description of mathematical intimacy underlines its focus on the profound relation between the individual and mathematics.

In addition, Goldin (2008) claims that mathematical engagement, a form of mathematical intimacy, may be connected to flow. This connection is based on items, such as loss of self-consciousness while being highly engaged in problem solving, altered perception of time and experiencing satisfaction or enjoyment. But, in addition to these associations, the mathematical intimacy and flow analogy could include the challenge-skill balance, clear goals and intense concentration. Experiencing mathematical intimacy invokes a challenge-skill balance since under conditions of anxiety or boredom mathematical intimacy could not come to fruition. Becoming intimately engaged in solving math problems implies one possible clear goal of solving the problem, and furthermore assumes a certain level of concentration. Mathematical intimacy may lead to positive outcomes such as confidence in personal abilities to continue future problem solving activities, perseverance in pursuing solving math problems, or willingness to take risks due to a sense of safety provided by mathematical intimacy (DeBellis, 1998). Mathematical intimacy may also lead to negative outcomes, such as frustration, disappointment, or anger due to unexpected outcomes while solving math problems (DeBellis, 1998).

Enjoyment, an essential component of mathematical intimacy, is related to flow in doing mathematics (Seifert, Radu, & Doyle, 2010). Flow experiences of mathematics students are similar to flow experiences of musicians or skateboarders (Seifert & Hedderson, 2010). However, some important differences emerge.

For mathematics students, experiencing flow is a deeply cognitive experience. And the combination of competence, challenge and concentration is central to experiencing flow. For example, students reportedly view mathematics as an enjoyable experience since solving problems make them feel good, and a sense of fulfilment or satisfaction prevails upon arriving at the solution to the problem. This, in turn, reinforces perceptions of competence in the subject. For such students, flow is commonly experienced alone, usually in an environment which they have control over. Engaging in solving challenging math problems becomes students' foundation for experiencing enjoyment. The structure of this discipline appeals to students and feeds their enjoyment. Mathematics has a set of clear rules that subsequently gave students a sense of control over their work and solutions. Within this description of flow, the centrality of challenge emerges (Seifert, Radu, & Doyle, 2010). Challenging problems allow students to become creative, and to experience loss of self-awareness and loss of surroundings. Within such settings, crystallizing thoughts into clear, incisive, swift and multi-layered thinking patterns stream from students' profound concentration.

Method

Participants in this study came from three classes of students enrolled in a first year mathematics course on Algebra and Trigonometry. The course is a prerequisite for Calculus courses, and students who take the course have failed to achieve the cut-off score needed to enrol in Calculus on a mathematics skills screening test. Two classes were offered in lecture-only format. Of the 69 students in the first class, 40 agreed to participate. Of the 66 students in the second class, 41 agreed to complete the survey. The third class was comprised of students enrolled in a blended version of the course, which combined lectures with participation in MyMathLab. Of the 72 students in this class, 29 agreed to participate. In total, data from 108 students were included in the analyses; 2 students were excluded because of missing data.

The course curriculum includes sections on real numbers, functions (e.g. exponential, logarithmic), trigonometry, analytic trigonometry, and polynomials. The standard course layout involves four hours of face-to-face lectures per week, written homework, midterms and final exams, and no computer mediated teaching and learning. The instructor of the blended class used a combination of online and face-to-face teaching methods in the four-hour time frame per week. In the first two weeks of classes, computer laboratories were held and the instructor guided students throughout the MyMathLab registration process and explained the features of the software. Students were expected to complete their weekly quizzes via MyMathLab's online setting. When MyMathLab was initially implemented, computer laboratories were in place and students attended them on a weekly basis (Kondratieva & Radu, 2008). Within this study, the lab instructor went to class once a week for about 20 minutes. During that time, the lab instructor clarified the math examples where most common mistakes occurred in the previous week's e-homework, and used MyMathLab help files that show how to correctly obtain the solution to a problem. Thus, the objective was to show students how they could get help from the software when they worked on their MyMathLab homework and studied for tests.

Early in the semester, students in both groups (lecture-only and blended format) completed a 64-item survey assessing several constructs related to affect and flow in mathematics. Some items used in the survey were adapted from the research work of Galbraith and Haines (1998), Tapia and Marsh (2004) and Cretchley (2008). For the purposes of this study, only those items assessing mathematical intimacy, determination (which was a composite of confidence and persistence), and flow were used. The intimacy construct was operationalized using items that asked students about feelings they have when doing mathematics. The items were in 4-point Likert format (strongly disagree, disagree, agree or strongly agree), and were subsequently dichotomized (disagree or agree) for analyses. The determination construct was operationalized using eight 4-point Likert items that asked students about their perceived ability to solve mathematics problems (see Figure 1). Flow was assessed using a nine-item scale similar to the Short Flow Scale (Jackson, Martin & Eklund, 2008). Each item, taken from the Dispositional Flow Scale-2 (Jackson & Eklund, 2004), assessed one of the nine dimensions of the flow construct (Csikzsentmihalyi, 1990) using a 5 point Likert format; items were summed to establish a total flow score.

Items pertaining to intimacy and determination were subjected to factor analyses using MPlus (Muthén & Muthén, 2006). A number of models were explored including a single factor model and a second-order factor model. These models were rejected because they did not provide a better fit to the data than a two-factor model. The first factor in this model represented mathematical intimacy and was characterized by item responses reflecting excitement, curiosity and enjoyment. The second factor was labelled determination and was characterized by confidence and persistence. As depicted in Figure 1, there was a sizeable correlation between intimacy and determination. The fit indices for the final model were adequate suggesting the model fit the data well (c^2 (47)=63.57, p=.05; CFI=.974; TLI=.980; RMSEA=.057).

The next step of the analysis was to determine if the two groups (lecture-only and blended format) possessed similar characteristics. Several multi-group analyses were undertaken to test the equality of means and variances between the two groups (Muthén & Muthén, 2006). In all instances, various models did not differ statistically from the equal means and equal variances model. This suggests that the lecture-only and blended classes had similar distributions on the intimacy and determination factors, as illustrated in Figure 2. However, the multi-group analyses indicated that the variance of the flow variable was not statistically different between the two groups, but the means were. Students in the blended classroom reported lower flow scores for mathematics than those in the lecture-only group, and the difference was substantial (ES=.62).

At the end of the semester, those students in the blended classroom completed the survey a second time, but with some modifications. Rather than asking about experiences with mathematics, the wording of the items was slightly altered to ask about intimacy, confidence and flow items asked about experiences in doing mathematics using MyMathLab. For example, factor structures for mathematical intimacy, determination and flow obtained in the analysis of the first survey were then imposed on responses to relevant items about their MyMathLab experiences.

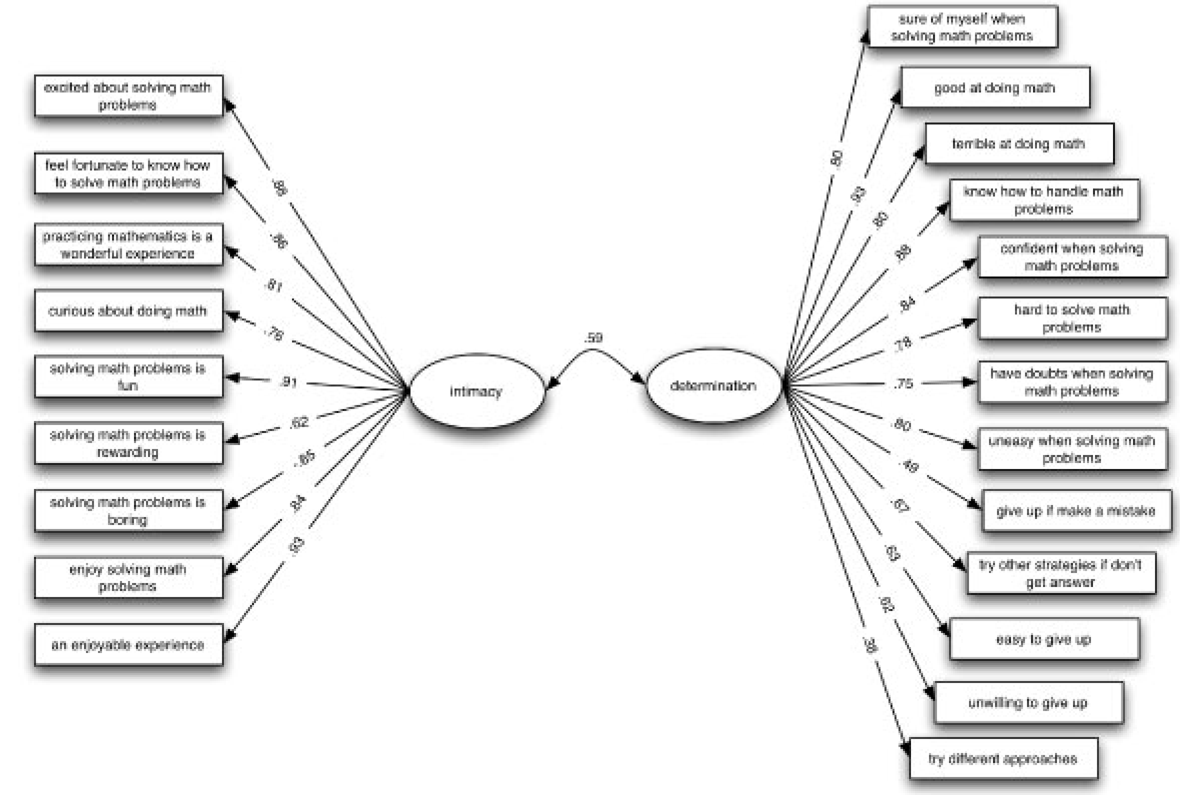

Figure 1. Factor structure for mathematical intimacy and determination.
Coefficients are standardized regression coefficients using probit regression.

Results

The mathematical intimacy factor structure from survey one was imposed upon the data for the students in the blended classroom, and multi-group analyses were conducted on the intimacy, determination and flow factors. Descriptive statistics and correlations among the factors are presented in Table 1. The table of correlations suggests that flow, intimacy and determination are strongly related. This may be indicative of a pattern of affect and behaviour in which feelings of intimacy are closely tied up with confidence and persistence, as well as flow-like experiences.

The results of the multi-group analysis were consistent across all three factors: equal means with unequal variances. Responses to the time 1 measure had less variance than those for the time 2 measure of intimacy (c^2 (16)=16.30, p=.50; CFI=1.0; TLI=1.0; RMSEA=.00), confidence (c^2 (26)=24.38, p=.55; CFI=1.0; TLI=1.0; RMSEA=.00) and flow (c^2 (1)=.50, p=.48; CFI=1.0; TLI=1.0; RMSEA=.00). There was greater variability in the range of scores for students' MyMathLab experiences than for mathematics in general, which is evident in both Table 1 and Figure 3.

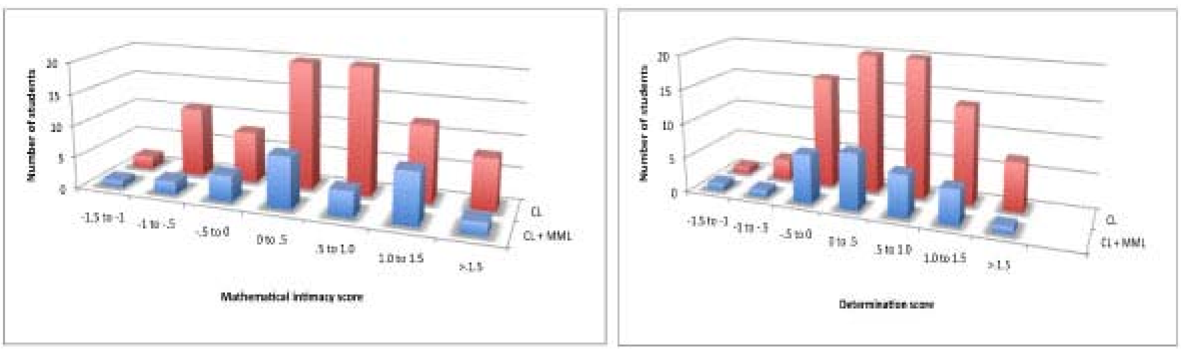

Figure 2. Distributions of mathematical intimacy and confidence scores by group.

The greater variability in time 2 scores is due to a trend towards lower scores in the time 2 measure than time 1. As illustrated in Figure 3, there is an apparent increase in the number of students having scores at the lower end of the scale, and it appears that this increase is due to a decrease in scores in the middle of the distribution. There is, however, a slight increase in the number of students at the upper end of the scale for both the intimacy and confidence factors.

Table 1 Descriptive statistics and correlations among factors

	Lecture only		**Blended class (Math in general)**		**Blended class (MyMathLab experience)**		**Correlations**		
	Mean (N=89)	**SD**	**Mean (N=29)**	**SD**	**Mean (N=29)**	**SD**	**Flow**	**Intimacy**	**Determin.**
Flow	26.58	4.32	23.93	4.15	22.93	6.18	1.0		
Intimacy	-.04	.71	.02	.71	-.20	.8	.57	1.0	
Determination	.04	.66	-.14	.69	-.10	.86	.59	.67	1.0

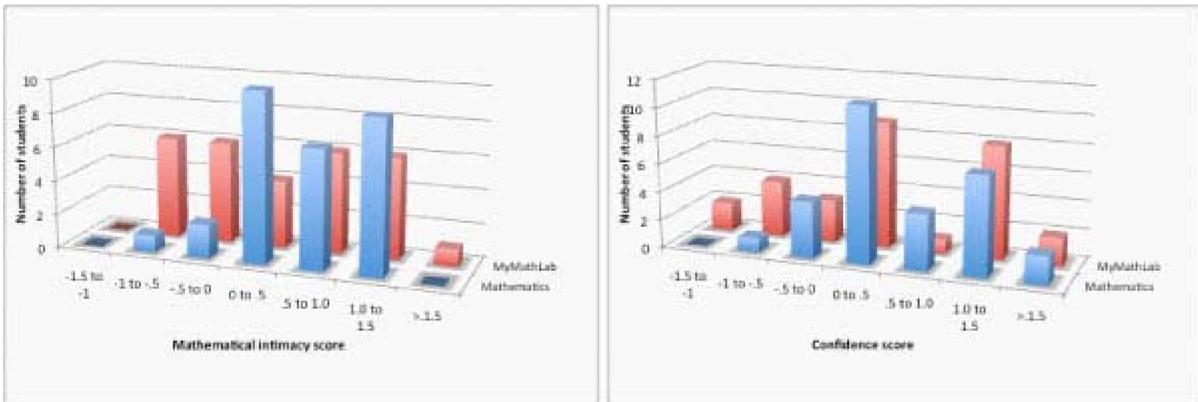

Figure 3. Distribution of mathematical intimacy and confidence scores for students in the blended learning group.

Discussion

In this study, students in a blended learning environment responded to two sets of items assessing mathematical intimacy, confidence, persistence and flow. The first set of items asked about mathematics in general; the second asked students about these constructs in the context of MyMathLab. Students in the blended class had mathematical intimacy scores similar to students in the lecture group. However, the mathematical intimacy scores for many students in the blended class were lower for MyMathLab than for mathematics in general, as indicated by an increase in range and variance. A similar finding was found for determination, but not flow.

Slightly lower mathematical intimacy scores for many students in the MyMathLab framework might be interpreted as a result of their inability to create a close bond with mathematics, and to experience joy and excitement in doing mathematics in the brief period of time when they were solving math online. Mathematical intimacy may foster the appearance of positive outcomes, such as confidence and perseverance (DeBellis, 1998). As such, for many students in the blended class, lower scores of mathematical intimacy translate into a decreased enjoyment and sense of well being that subsequently leads to poorer confidence, and thus slightly lowered confidence scores. But, perseverance alongside confidence is viewed as possible outcome of mathematical intimacy (DeBellis, 1998), as is indicated by the relatively high correlation among the factors. However, similar scores in determination between the two groups might be explained through students' devotion in passing the course or in obtaining a good grade regardless of mathematical context: blended or traditional. Based on the theoretical similarities of the mathematical intimacy and flow (Goldin, 2008), we expected that students would have similar experiences and scores. It came as a surprise that even if mathematical intimacy scores were comparable in the blended and lecture-format settings with respect to general math, there was a substantial statistical difference in flow scores, as the students in the blended classroom reported lower flow scores in general math than those in the lecture only class setting. However, there was no change overall in the two surveys.

There are a number of limitations to this study that suggest further investigation is warranted. First, the measures may not have been in sufficient temporal proximity to their activities to accurately measure students' affect. For example, the flow items asked about flow characteristics in general, and not about a specific activity at a given moment, and not capturing students' inner experiences (Hurlburt & Heavey, 2006). Second, the study may have been limited by low statistical power to detect differences. The small sample size and the lack of a true pre-test post-test design reduced statistical power to detect differences.

However, the study suggests that MyMathLab may have a positive effect for some students, and a negative effect for others. A mixed-methods study employed pre-test post-test design with either case studies or interviews could shed much more light on students' inner experiences when using online technology for learning mathematics.

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